

Optimum Design of Space Tensegrity Dome

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Abstract— *Tensegrity structure is a special type of pin jointed frame (truss), in which the tension members were replaced by cables. It can be defined as (a compression members in sea of tension). This type of structures features by light weight, covering large span, self-stressed, and it is deployable. This paper sheds light on the tensegrity structures of several destinations historical, benefits, usage, analysis and design. Also a comparison between tensegrity dome structures and trussed dome structures is presented, for 100 m dome span. When the effects of change of some parameters as number of tangential and radial division, inclination of upper and diagonal chord is studied. Also the searching for the optimum case was done.*

Index Terms—Analysis, Design, optimization, tensegrity structures, domes

I. INTRODUCTION

Throughout the ages the structural engineer is searching for stable structures, which give the owner's requirements without exceeding the allowable limits of design with minimum cost. Through his research for development types of structures he discovered a new structural types, one of them is the truss structure in which he can release it from bending moment and shear force due to its pin jointed connections. So the truss members are designed only for buckling and axial force. But in the most skeleton structure the buckling play an important role in choosing the cross section area, although the axial force may be very small the cross section may be large due to buckling. So he search for other structure, in which he can overcome this problem (buckling) he discover a new type of structures called tensegrity structures. In tensegrity structures the member has one type of axial force compression force or tension force for all different design load cases, not as the truss where the member may be has compression force or tension force according to the load case. Then the tension members can be replaced by cables which have strength greater than steel strength and it didn't affected by buckling, the number of compression members is small in comparison with the tension members. The feature that the members have one type of axial force is coming from the form finding process, which is the searching for the joint locations and the member pretension. So it can be said that the stability of tensegrity is coming from its self-stress. This type of structures features by light weight, covering large span, self-stressed and it is deployable. Carstens, S and Kuhl, D.,^[1] mentioned that the tensegrity is an artificial word, composed of the two expressions tensional and integrity. The American architect Richard Buckminster Fuller and his

student Kenneth Snelson are the inventors of the tensegrity idea. Schenk, M.,^[2] mentioned that, the first true tensegrity structure is usually attributed to the artist Kenneth Snelson, who created his X-piece sculpture in 1948 "Fig.1". Although some people point to 1921 structure which studied in balance by Russian constructivist K.Ioganson as prior art the matter is clearly only of historical interest. Buckminster Fuller and Snelson contributed most to conception of tensegrities. Motro, R.,^[3] presented a lot of subject concern with tensegrity he care of the history of tensegrity and definition and concept. He stated that Richard Buckminster Fuller described the tensegrity principle as "islands of compression inside an ocean of tension". A. Pugh defined the tensegrity as: "A tensegrity system is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space." Motro, R., suggest this definition: "A tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components."

Form finding process one of most important topic of tensegrity structures a lot of researches deal with this point early and still under development, and it is one of characteristic which make the tensegrity differ than other structures. Tibertand, A.G and Pellegrino, S.,^[4] classified this methods into two main categories, kinematical methods which include (Analytical Solutions, nonlinear programming, and dynamic relaxation), and statically methods which include (Analytical Solutions, Force Density Method, energy method, and reduced coordinates). Charalambides, J. E.,^[5] developed a computer based utility that will facilitate the design professional to devise, and construct a specific morphological variation of tensegrity structure systems. CHANG, Y. K.,^[6] developed a new form finding method which is based on genetic algorithm methodology. Murakami, H.,^[7] large deformation kinematic and kinetics were presented in both Eulerian and lagrangain formulation. Murakami, H.,^[8] linearized lagrangain equation developed in^[7] were employed for static analysis of cyclic cylindrical tensegrity modules, linearized equilibrium equation at natural configuration were used to investigate initial shape. Bel Hadj, N and Rhode, L and Smith, I. F. C.,^[9] presented a modified dynamic relaxation (DR) algorithm for static analysis and form-finding. Gustavo, P and Carlos, F and Moreira, L and Vladimiro, E and Greco, M.^[10] presented the experimental analysis of a tensegrity structure based on truncated icosahedron geometry. The objective of the study is to characterize the mechanical behavior of a tensegrity dome.

They found that Based on the experimental analysis, the structural behavior is characterized mainly by geometrical nonlinearity. From the strains of the bars and the strains of the cables, the behavior of the materials is assumed as linear. The strains were small for the aluminum bars. Therefore, for the applied force it can be considered that the material was in the proportional phase during the experimental tests, the top of the structure rise and then go down with the rest of the structure. This behavior occurs due the pre-stress that acts in the cables. Liapi, K. A., [11] present and discusses the configuration of new tensegrity modules and structures for the Hellenic Maritime museum. Van Telgen, M.V., [12] investigated the problems that may exist in the design of circular and elliptical tensegrity domes and compression hoops.



Fig. 1 SNELSONS X-piece sculpture in 1948 [2].

II. METHODOLOGY AND PROGRAMMING

The Design of tensegrity structures is divided into three stages; first one is the form finding process in which the joint locations and the members' pretension are calculated. Secondly, the process of analysis is coming to determine the final axial force in members. Thirdly, the cross section area of structure members is calculated in the last stage. In this part, the methods used in these stages are represented, in addition to the loads on the structures and material assumptions. Finally a programming is presented.

A. Form finding:

To get the form finding of dome, the analytical method is used, where the tensegrity dome shape is derived starting with simple beam until the dome reached its final shape, which shown in Fig. 2. [12]

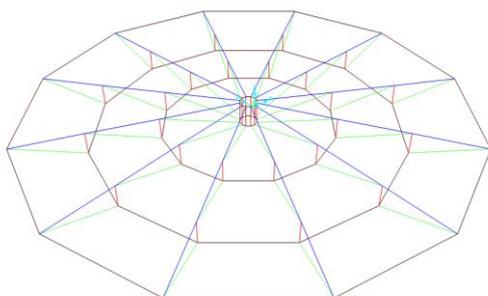


Fig. 2 tensegrity dome

Note that it is not the only geometry and topology of tensegrity dome, but that is an example. The designer chooses the inclination of upper chord and diagonal chord. Also he chooses the number of radial and tangential division (Fig. 2). To get the pretension in members, the equilibrium at all nodes are used when a half planar tensegrity of the dome shown in Fig. 2 was taken as shown in Fig. 3.

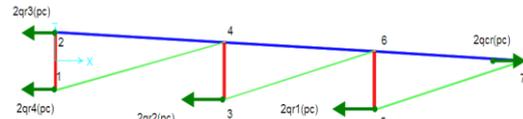


Fig. 3 half planar tensegrity of the dome

Now if the pre-compression in the outer compression ring (qcr) is assumed, and take its planar component $2qcrPc$, and study the equilibrium of joint (7) in both X and Z directions, the pretension in both upper and inclined cables can be obtained, then by studying the equilibrium of joint (5), the pretension in vertical strut and radial cable can be gotten, then studying the equilibrium of joint (6), and so on for all joints to get all the pretension in all members. Note that all members' pretension can be multiplied or divided by factor. Also the outer compression ring can be made from pre-stressed reinforced concrete, or replaced by cable to release the moment on the column as shown in "Fig. 4".

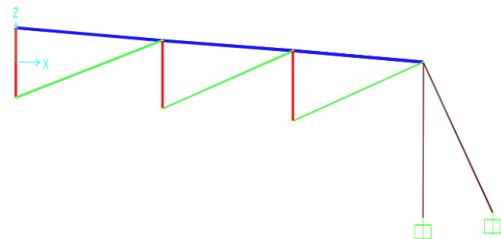


Fig. 4 Replacement of Compression Ring by Cable.

B. Loads and material assumptions

In this study the tensegrity domes is design only due to vertical loads, which have the largest effect in the determination of the cross section area of members, and the vertical displacement of dome. This vertical loads come from the tensegrity dome own weight, covering and the live load. These loads can be calculated as follow:

$$W = W_{\text{own weight}} + W_{\text{covering}} + W_{L.L} \quad (1)$$

$$W_{\text{own weight}} = \sum_1^n 7.85 * L * A \quad (2)$$

$$W_{\text{covering}} = 5 - 20 \text{ kg/m}^2$$

$$W_{L.L} = 60 - (200/3) * \tan \lambda \text{ kg/m}^2 \quad (3)$$

Where:

L: member length.

n: number of total members.

A: member cross section area.

λ : inclination of inaccessible roof

The vertical loads which acting on every joint can be calculated by multiply the load intensity by the area surrounded this joint as shown in Fig 5.

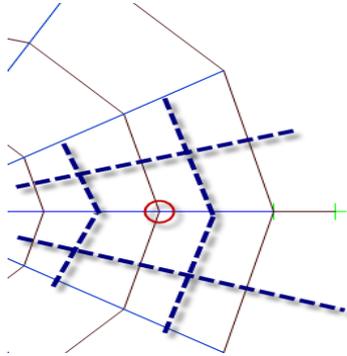


Fig. 5 The area surrounded of one joint of the dome

C. Material assumptions:

The strut material (steel 37) characteristics are assumed according to Egyptian steel code^[13] as shown in table 2.1. While the cables material characteristics are assumed according to Harazaki, et. al,^[14] as shown in table 2.2.

Table .1: The strut material characteristics (st. 37):

Mass Density	7.85 t/m ²
Modulus of Elasticity	2100 t/cm ²
Yield strength	2.4 t/cm ²
Ultimate strength	3.6 t/cm ²

Table 2: The cables material characteristics:

Mass Density	7.85 t/m ²
Modulus of Elasticity	2000 t/cm ²
allowable strength	8.2 t/cm ²
Ultimate strength	18t/cm ²

D. Analysis of tensegrity structures:

The assembly stiffness method is used to analyze the loaded tensegrity structures to get the internal force in members.

Stiffness matrix of tensegrity member:

Due to the pre-stress in tensegrity members the stiffness matrix is consisting of two terms, linear elastic stiffness matrix (Ko), and geometric stiffness matrix (Kg). The global tangent stiffness matrix (Kt) of one member can be calculated as follow⁽¹⁵⁾:

$$K_t = T * K_o * T' + K_g \tag{4}$$

Where

$$K_o = \begin{bmatrix} G & 0 & 0 & -G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$K_g = \begin{bmatrix} q & 0 & 0 & -q & 0 & 0 \\ 0 & q & 0 & 0 & -q & 0 \\ 0 & 0 & q & 0 & 0 & -q \\ -q & 0 & 0 & q & 0 & 0 \\ 0 & -q & 0 & 0 & q & 0 \\ 0 & 0 & -q & 0 & 0 & q \end{bmatrix} \tag{6}$$

Where:

G: modified axial stiffness ($G = EA/L - q$)

q: member force density.

T: transformation matrix.

After calculating the tangent stiffness matrix for all members, the structure stiffness matrix can be constructed by assembly.

E. Internal force:

The member internal force consists of two terms, the member elongation and the bar pre-stress.

$$N = EA / L * e_b + q \tag{7}$$

Where:

N: member axial force.

e_b: member elongation.

It must be noted that the normal force of cable members must be positive else the pretension of cables must be increased by multiplying it by factor for all members and reanalysis the structure.

F. Design of members' cross section:

To determine the cable (tension members) cross section, the stress equation can be used as follow:

$$A_{cable} = F_{cable} / \sigma_{allowable(cable)} \tag{8}$$

While design of tensegrity compression members (struts) obey the Egyptian steel code⁽¹³⁾ as follow:

The allowable stress in axial compression (Fc) for t < 40 mm:

a) If $\lambda < 100$
 $F_c = 1.4 - 0.000065 \lambda^2 \tag{9}$

b) If $\lambda > 100$
 $F_c = 7500 / \lambda^2 \tag{10}$

Where:

λ : slenderness ratio = $kL/r < 180$.

k: buckling length factor.

r : radius of gyration.

L: strut length.

$$A_{strut} = F_{strut} / F_c.$$

G. Programming:

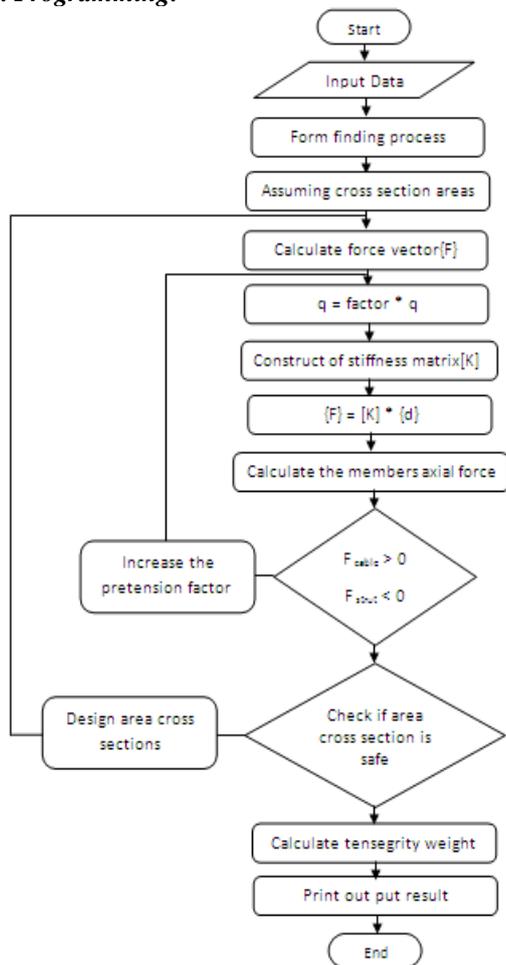


Fig. 6 Flow chart of analysis and design of tensegrity program

III. RESULTS

The 100 m span dome in this study is divided into number of radial divisions as 3, 4, 5 and 6 and number of tangential divisions as 12, 14, and 16. The inclination of upper chord is between 2 and 6 degrees, and the inclination of diagonal chord is between 15 and 25 degrees. The tensegrity structure has an advantage of reduced weight when compared with the corresponding truss structure. This study focuses on the percentage of tensegrity structure weight to the corresponding truss structure weight (TTWR factor) due to change of number of radial and tangential divisions at constant inclination of upper and diagonal chords. In last part, the optimum cases of this dome were clarified by tables and charts.

A. Parametric study:

1- Chord inclination ($\theta_1=2, \theta_2=15$):

Figure 7 shows the relation between TTWR factor and number of radial divisions. It is shown that the TTWR ranges from 0.3 to 0.6 which clarify the benefit of the tensegrity structure by decreasing the weight of the structure. It is shown that the TTWR factor decrease with increasing the number of radial divisions, because the

increase of radial divisions leads to decrease in tensegrity weight by very small values as shown in figure 8, because increasing of radial divisions leads to the decrease in members length which leads to the decrease in members force which coming from pre-tension ($q=F/L$). But in the same time it increases the number of members which represents weights, so the tensegrity weight decreases by small values. The decreasing of tensegrity structure weight not absolute, but it reverses at specific number of radial divisions which differ according to tensegrity parameters (number of tangential divisions, upper and diagonal inclination angles), that because large increasing in number of radial divisions increases the weight coming from unnecessary increasing in number of members by percentage larger than the decreasing coming from shorting in member length and decreasing in its pre-tension, that clarified in figure 7 and 8 for tangential divisions equal 16 and radial division equal 6. Also increasing number of radial divisions leads to increase in truss weight by large values due to unnecessary increasing in number of members which represents weights as shown in figure 9. It is shown also that the TTWR decreases with increasing the number of tangential divisions due to the decrease in ring members length which leads to decrease in members force which coming from pre-tension ($q=F/L$), also decreases the force component from radial cables in planar stability due to increasing the angle between radial cables and planar members. The optimum case occurs at T-divisions (tangential) equal 18 and R-divisions (radial) equal 6.

2- Chord inclination ($\theta_1=2, \theta_2=20$):

Figure 10 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 20 degree. It is shown that the TTWR ranges from 0.35 to 0.7. It is shown also that the TTWR factor decreases with increasing the number of radial divisions. It is shown also that increasing the diagonal chord inclination ($\theta_2=20$) increases the TTWR factor due to the increase of the tensegrity structure weight as shown in figure 11, because the diagonal cables need more pre-tension to achieve stability with radial cables due to decreasing its horizontal components in planar stability this increasing in pre-tension also leads to increasing upper chord pre-tension which leads to increase in final axial forces of members and cross section areas which represents tensegrity weight. Also increasing diagonal inclination angles leads to decrease of truss structure weight as shown in figure 11, so the TTWR increases with increasing the diagonal inclination angle. The optimum case occurs at T-divisions equal 24 and R-divisions equal 6.

3- Chord inclination ($\theta_1=2, \theta_2=25$):

Figure 12 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 25 degree. It is shown that the TTWR ranges from 0.4 to 0.75. It is shown that the TTWR

factor decrease with increasing the number of radial divisions. It is shown also that increasing the diagonal chord inclination increases the TTWR factor due to the increase of the tensegrity structure weight and the decrease of truss structure weight. The optimum case occurs at T-divisions equal 24 and R-divisions equal 6.

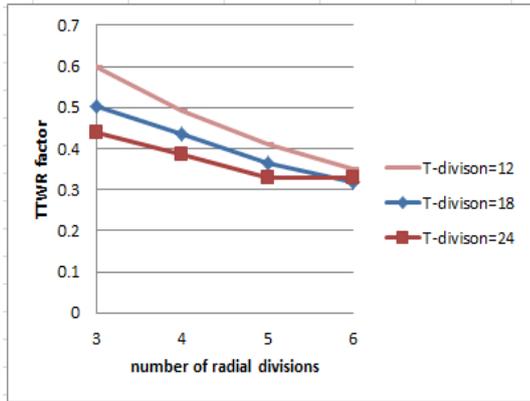


Fig 7: Relation between TTWR factor and no. of radial division for ($\theta_1=2, \theta_2=15$)

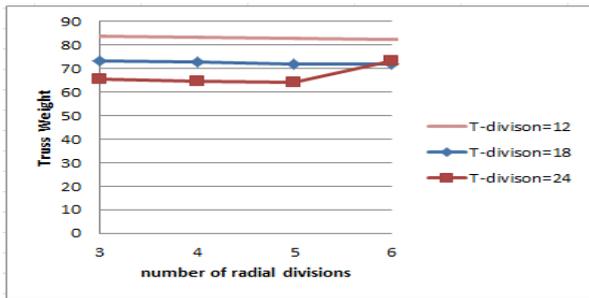


Fig 8: Relation between tensegrity weight and number of radial division for ($\theta_1=2, \theta_2=15$)

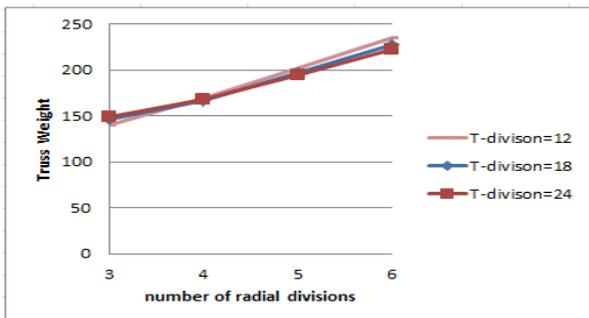


Fig 9: Relation between truss weight and number of radial division for ($\theta_1=2, \theta_2=15$)

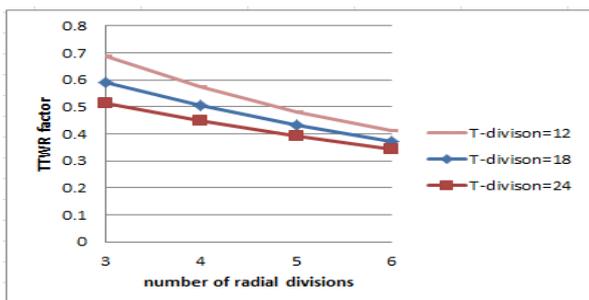


Fig 10: Relation between TTWR factor and no. of radial division for ($\theta_1=2, \theta_2=20$)

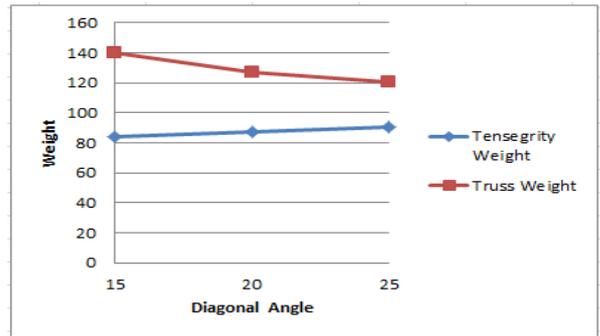


Fig. 11: Relation between tensegrity and truss weights and diagonal inclination angle at upper angle equal 2 and R-division=3, T-division=12.

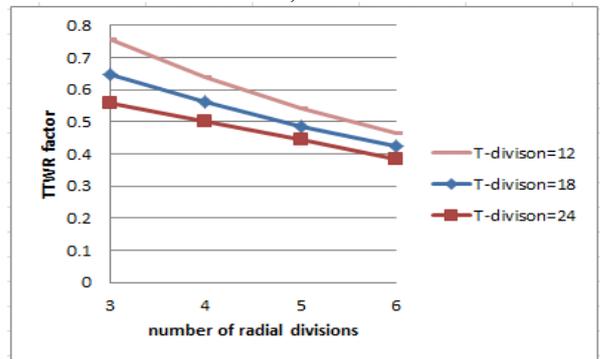


Fig 12: Relation between TTWR factor and no. of radial Division for ($\theta_1=2, \theta_2=25$)

4- Chord inclination ($\theta_1=4, \theta_2=15$):

Figure 13 shows the relation between the TTWR factor and number of radial divisions when the upper inclination angle increases to 4 degrees. It is shown that the TTWR ranges from 0.3 to 0.6. It is shown that increasing the upper chord inclination slightly increases the TTWR factor due to the increase of the tensegrity structure weight because the upper chord cables needs more pre-tension to achieve stability due to decrease its horizontal component in planar analysis. This increase in upper chord pre-tension leads to increase the pre-tension in diagonal chord, which leads to large final axial force and large tensegrity weight. Also, the increase of upper inclination angle leads to increase in truss structure weight, with approximately equal rate as tensegrity structures as shown in figure 14. It is shown also that the TTWR factor decreases with increasing the number of radial divisions. The optimum case occurs at T-divisions equal 24 and R-divisions equal 6.

5- Chord inclination ($\theta_1=4, \theta_2=20$):

Figure 15 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 20 degree. It is shown that the TTWR ranges from 0.37 to 0.7. It is shown that the TTWR factor decreases with increasing the number of radial divisions. It is shown also that increasing the diagonal chord inclination increase the TTWR factor due to the increase of the tensegrity structure weight and the decrease of the truss

structure weight. The optimum case occurs at T-divisions equals 24 and R-divisions equal 6.

6- Cord inclination ($\theta_1=4, \theta_2=25$):

Figure 16 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 25 degree. It is shown that the TTWR ranges from 0.42 to 0.75. It is shown that the TTWR factor decreases with increasing the Number of radial divisions. It is shown also that increasing the diagonal chord inclination increase the TTWR factor due to the increase of the tensegrity structure weight and the decrease of the truss structure weight. The optimum case occurs at T-divisions equal 24 and R-divisions equal 6.

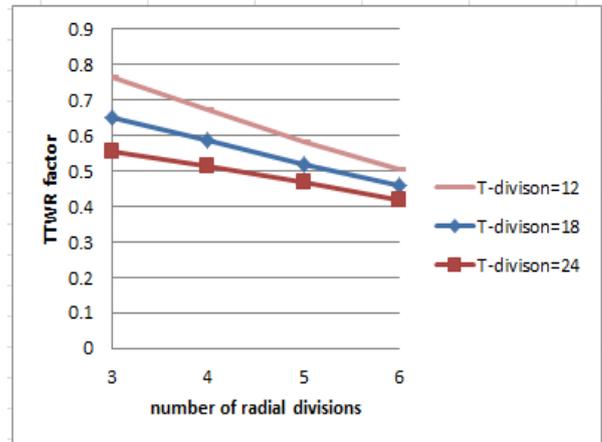


Fig 16: Relation between TTWR factor and number of radial division for ($\theta_1=4, \theta_2=25$)

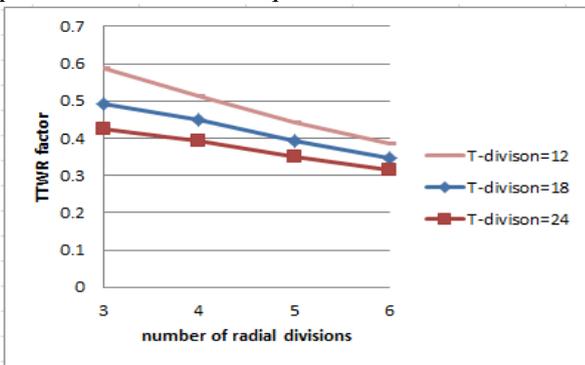


Fig 13: Relation between TTWR factor and number of radial division for ($\theta_1=4, \theta_2=15$)

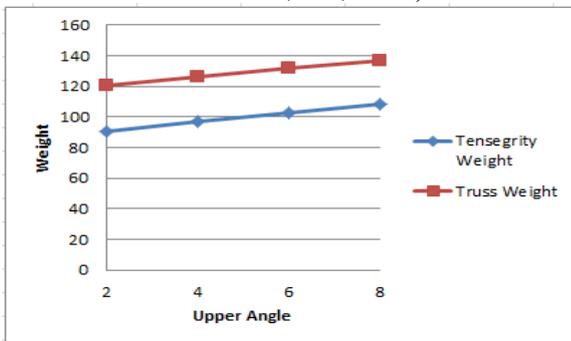


Fig 14: Relation between tensegrity and truss weights and upper inclination angle at diagonal angle equal 25.

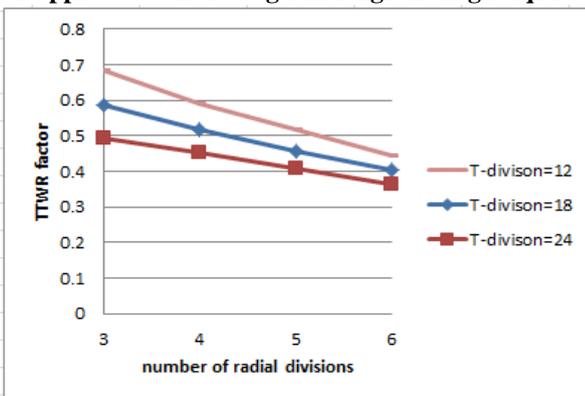


Fig 15: Relation between TTWR factor and number of radial division for ($\theta_1=4, \theta_2=20$)

7- Chord inclination ($\theta_1=6, \theta_2=15$):

Figure 17 shows the relation between the TTWR factor and number of radial divisions when the upper inclination angle increases to 6 degrees. It is shown that the TTWR ranges from 0.35 to 0.65. It is shown increasing the upper chord inclination increase the TTWR factor due to the increase of the tensegrity structure weight and truss structure weight with approximately equal rate. It is shown also that the TTWR factor decreases with increasing the number of radial divisions. The optimum case occurs at T-divisions equal 24 and R-divisions equals 6.

8- Chord inclination ($\theta_1=6, \theta_2=20$):

Figure 18 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 20 degree. It is shown that the TTWR ranges from 0.4 to 0.7. It is shown that the TTWR factor decreases with increasing the Number of radial divisions. It is shown also that increasing the diagonal chord inclination increase the TTWR factor due to the increase of the tensegrity structure weight and the decrease of the truss structure weight. The optimum case occurs at T-divisions equals 24 and R-divisions equal 6.

9- Chord inclination ($\theta_1=6, \theta_2=25$):

Figure 19 shows the relation between the TTWR factor and number of radial divisions when the diagonal inclination angle increases to 25 degree. It is shown that the TTWR ranges from 0.45 to 0.8. It is shown that the TTWR factor decreases with increasing the Number of radial divisions. It is shown also that increasing the diagonal chord inclination increase the TTWR factor due to the increase of the tensegrity structure weight and the decrease of the truss structure weight. The optimum case occurs at T-divisions equals 24 and R-divisions equal 6.

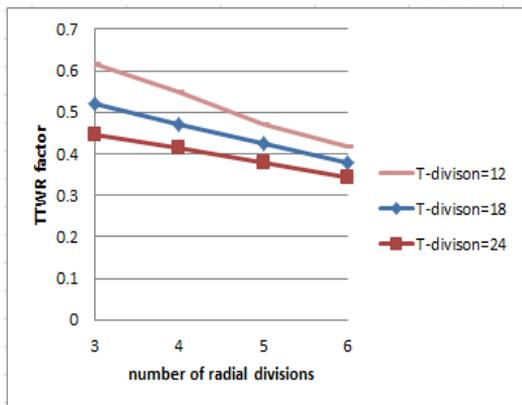


Fig 17: Relation between TTWR factor and number of radial division for ($\theta_1=6, \theta_2=15$)

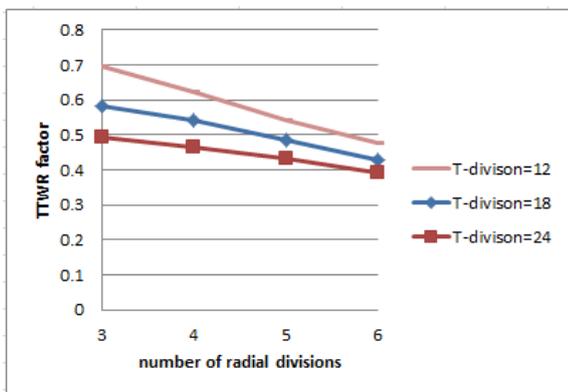


Fig 18: Relation between TTWR factor and number of radial division for ($\theta_1=6, \theta_2=20$)

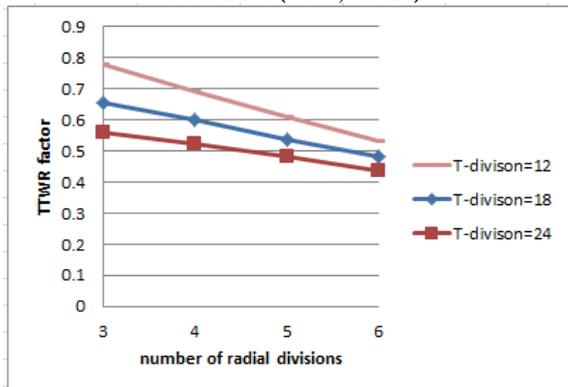


Fig 19: Relation between TTWR factor and number of radial division for ($\theta_1=6, \theta_2=25$)

B. the optimum Domes

Table (3) the optimum dome parameters and weight for 100 span dome:

type	100 m
T-division	16
R-division	5
θ_1	2
θ_2	15
Weight (ton)	64
Tens/Truss	0.53

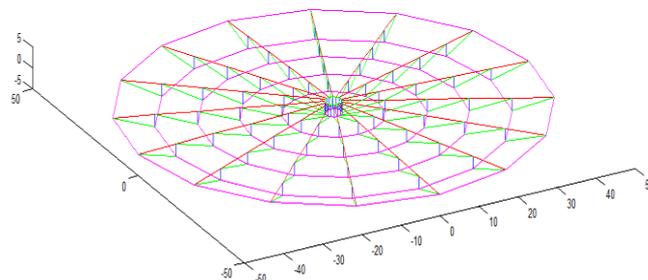


Fig 20: The optimum tensegrity Dome for 100 m span

IV. CONCLUSIONS

A MATLAB program is used in this study to make a comprehensive program (form finding, check tensegrity condition and stability, analysis and design) to study the tensegrity structures. The objective function in this study is structure weight which means the structure cost, which is the focus of the owner in most cases, and the aim of the engineer. 100 m span dome was studied with its parameters which are the number of radial divisions, number of tangential divisions, inclination of upper chord and inclination of diagonal chord. Finally a comparison between it and the trussed dome was done. Many points can be concluded from this research:

- 1- In tensegrity dome structure the best number of radial division is between five and six divisions.
- 2- The high number of tangential divisions is preferred for tensegrity dome structures.
- 3- In tensegrity dome structures the small inclination angles for both upper and diagonal chord is the best.
- 4- The best solution to decrease the vertical displacements to achieve the allowable displacements is increasing the pretension in the members of tensegrity dome.
- 5- Using tensegrity structures in covering (domes) can save 50 % from the cost.

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